THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 57, No. 5, May–June 1978
Printed in U.S.A.

Nonlinear Analysis of a Photovoltaic Optical Telephone Receiver

By D. A. KLEINMAN and D. F. NELSON

(Manuscript received February 2, 1977)

The general problem is considered of calculating the voltage when a sinusoidal current generator is connected to a parallel combination of a linear and a nonlinear resistance load. A practical algorithm is described for computing any desired moment and any Fourier component of voltage. An alternative approximate treatment is also presented which avoids numerical integrations and is valid when the nonlinear characteristic is rapidly varying. Both methods are applied to a photovoltaic optical telephone receiver employing a silicon nπp-photodiode and a conventional ring armature telephone receiver coupled by a transformer. Harmonic distortion is presented for several illustrative cases. A clipping level is defined for the receiver, and it is proposed that the receiver clipping level should be matched to the clipping level of the analog optical channel bringing the signal. On the basis of this principle a simple procedure is given, along with the necessary curves, for determining the required optical power at the source and the optimum transformer ratio for any value of transmission loss between source and receiver. An illustrative example is given for an analog dynamic range of 18 dB that requires a peak source power in the lightguide of 0.9 mW. This type of receiver may find limited application if lightguides ever serve customers directly.

I. INTRODUCTION

There is now strong indication[1,2] that optical transmission using lightguides[3] and optical cables is technologically approaching a readiness for use in telecommunications. While we are not here suggesting that any extensive application of optical telephones is foreseeable, we have nevertheless found the prospect of limited special applications sufficiently interesting to undertake a study of optical telephone receivers from a device point of view. The receiver is only one of a number of devices, some of which perhaps have not even been invented yet, that would

be required in an optical telephone. In addition, further devices would be required in the electrical-optical interface between the lightguides and the metallic network. The receiver, however, determines one key property of the system, the optical power required at the interface to transmit speech to the human ear with an acceptable volume and quality.

We presuppose that an optical telephone receiver is required to convert analog-modulated light power to sound pressure at the ear with no other power available. There are two mechanisms that might be employed to do this using analog modulation: the optoacoustic effect in which sound is directly produced when power-modulated light is absorbed, and the photovoltaic effect in which an intermediate electrical signal is produced which produces sound by way of an electrical earphone. We have previously completed a theoretical[4,5] and experimental[6] study of optoacoustic receivers in which it was necessary to solve a variety of linear acoustical problems to establish the feasibility of the device and to obtain its response. Nonlinear distortion was not considered and is not believed to be very important for optoacoustic receivers. Optimization in that case is a matter of maximizing the response subject to the requirement of a flat response over the telephone voice band, 300–3300 Hz.

Subsequently we have attempted to define an optimum photovoltaic receiver in a similar way on the basis of maximizing the linear response. It might at first appear that this is a simple problem, because a suitable photodiode and earphone are already available, the required frequency response has already been engineered into the earphone, and one might expect that it is only required to match the impedances of the photodiode and the earphone by a simple two-port network (e.g., a transformer). However, we have now concluded that no optimum of this type exists for the photovoltaic receiver, and a different principle of optimization is required which is based on the nonlinearity of the photodiode and the quality of speech reproduction required in the system. Thus it has not been possible to keep system concepts completely out of the discussion.

We propose here a very simple principle of optimization, which we call "quality matching," which leads to important conclusions about the optical power at the interface, the dynamic range of the system, sound levels in the system, and of course, the receiver design.

We base our discussion on the circuit of Fig. 1. Modulated light power u is conducted by a lightguide to the photodiode, which may be regarded as a current generator g in parallel with a junction current $j(v)$. The photodiode assumed is a specific silicon $n^+\pi p^+$ structure[7] designed and packaged for lightguide use with a light-sensitive area of diameter 80 μm and a quantum efficiency at 900 nm of about $\eta = 0.8$. It is ideal for

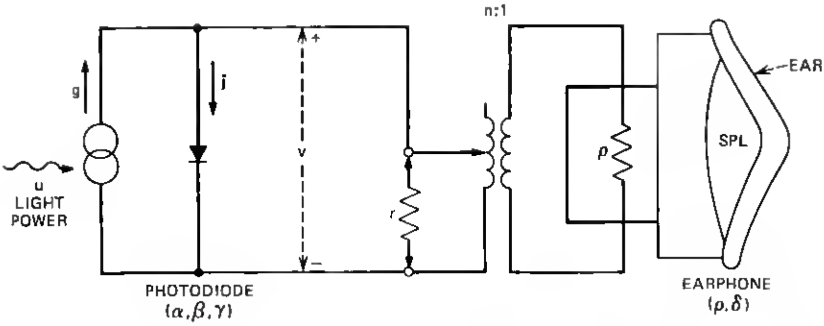

Fig. 1—Circuit of photovoltaic receiver. The earphone resistance ρ is transformed to r.

our purpose because of its low series resistance ($\approx 1\ \Omega$). The earphone is the ring armature telephone receiver[8] which provides good sensitivity with essentially flat response over the band 300–3300 Hz. The photodiode and earphone are coupled by an ideal transformer having a turn ratio n adjustable by virtue of primary taps. The size and cost of the transformer would be approximately proportional to the maximum value of n. The effective load resistance seen by the photodiode is zero at dc and $r = n^2\rho$ at all signal frequencies, ρ being the earphone resistance. The value of n is to be selected at the time of installation in accordance with our optimizing principle.

The first result to be described in this paper is the nonlinear analysis itself, in Section II. Mathematically our problem is the following: Find the periodic voltage response $\nu(t)$ to a source current $g_1 \cos \omega t$ in Fig. 1 assuming r is linear and independent of frequency (except dc), $j(\nu)$ is nonlinear and monotonic, and $\nu(t)$ has zero average value. What makes the problem awkward to treat by textbook methods[9] is the restraint on the average value. The analysis, so far as we know, is not covered in texts on nonlinear circuits, and may be applicable to a variety of situations. In Section III is given an approximate method called the clipping model which we have found quite reliable and especially appropriate for the photovoltaic receiver.

The main body of the paper consists of Sections IV, V, VI, and VII, devoted to the sensitivity, harmonic distortion, clipping level, and quality matching of the receiver, respectively. The sensitivity is an inverse measure of response (the smaller the better!) defined here in the same way as in our previous work as the amplitude of sinusoidal (power) modulation of u required to produce at the ear the average speech power level (81 dB SPL) found in the telephone network surveys. It is interesting to note that the minimum sensitivity is achieved for $r \approx 3 \times 10^5\ \Omega$, which is considerably smaller than the small signal resistance of the photodiode, $\approx 8 \times 10^9\ \Omega$. This shows the essential importance of the nonlinear analysis

presented here since a linear analysis would lead to the equality of these two resistances. The minimum sensitivity is not, in general, the optimum because of nonlinear distortion.

We have made extensive calculations of the second, third and fourth harmonic distortions and the total harmonic distortion. Some curves of total harmonic distortion for a few selected cases are presented in Section V. We have found it difficult, however, to draw concrete conclusions from a consideration of the harmonic distortion that could be used as the basis for an optimizing principle. Rather, we have turned to clipping as the most convenient, relevant, and useful way of specifying the nonlinear distortion.

The clipping model, Section III, is based on the assumption that the distortion of the waveform is an abrupt one-sided clipping of tbe peak. This assumption is shown to be an adequate modeling of the nonlinear effects of an exponential junction characteristic. It is shown in Section VI that a clipping level can he defined which is analogous (except for being one-sided) to the clipping level of an analog-modulation channel. Our optimizing principle, "quality matching," then follows in Section VII in an obvious way, namely that the clipping levels of the receiver and the analog light channel feeding the receiver should be set equal to each other. The receiver then has the minimum sensitivity consistent with the requirement that the quality of the channel not be degraded by the receiver. This means that the system could operate at the minimum power consistent with a given dynamic range. By using the curves given in Section VII, quality matching can be carried out to determine for any desired dynamic range the optical power required and the correct transformer ratio for the receiver needed for the particular value of the transmission loss at that receiver.

A summary with discussion and conclusions is given in Section VIII. This is followed by two appendixes containing useful background material on speech quality and the electrical-optical interface.

II. NONLINEAR ANALYSIS

In the circuit of Fig. 1 ignore the light, the transformer, and the earphone; assume that the effective load resistance r is constant at all signal frequencies of interest and zero at dc. Let the current generator be

$$\begin{aligned} g &= g_o + g_1 \cos\theta \\ \theta &\equiv \omega t \\ g_o &\geqslant g_1 \geqslant 0. \end{aligned} \tag{1}$$

Define averages over time by

$$\langle f(\theta) \rangle_\theta \equiv \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\theta) d\theta \equiv f_o. \tag{2}$$

The equation of the circuit is

$$\nu - rj_o = rg_1 \cos\theta - rj(\nu), \tag{3}$$

which implies the relation

$$\nu_o = 0. \tag{4}$$

Concerning $j(\nu)$ we only assume

$$j(\nu)' \equiv dj/d\nu \geqslant 0, \qquad j(0) = 0 \tag{5}$$

which rules out negative-resistance instabilities. We call $\nu(\theta)$ the solution and $\theta(\nu)$ the inverse solution.

Define the parameters

$$\nu(\pi) \equiv a, \qquad \nu(0) \equiv b, \tag{6}$$

and

$$q_\pm \equiv \frac{1}{2}\left[(b \pm a) + r(j(b) \pm j(a))\right]. \tag{7}$$

We can assume ν and θ are restricted to the domains

$$a \leqslant \nu \leqslant b, \qquad 0 \leqslant \theta \leqslant \pi \tag{8}$$

and the inverse solution $\theta(\nu)$ is unique. It follows that

$$\nu_o = a + \frac{1}{\pi}\int_a^b \theta(\nu)\, d\nu \tag{9}$$

and

$$j_o = j(a) + \frac{1}{\pi}\int_a^b j(\nu)'\theta(\nu)\, d\nu. \tag{10}$$

From (3) we obtain the inverse solution

$$\theta(\nu) = \cos^{-1}[(\nu + rj(\nu) - q_+)/q_-] \tag{11}$$

and the implicit relations which determine a,b

$$q_+/j_o = r \tag{12}$$

$$(q_-/q_+)j_o = g_1. \tag{13}$$

Our algorithm proceeds as follows: (i) choose initial values for a,b; (ii) compute $q_\pm$ from (7) and j_o from the integral (10); (iii) test (12) and (13); (iv) iterate this procedure with adjusted values of a,b until the desired precision is achieved; (v) the inverse solution is now given by (11) with the final values of a,b; (vi) as a check, evaluate ν_o from (9) and test (4). Since (9) and (10) involve numerical integrations, this algorithm requires a large computer.

A function of ν such as $j(\nu)$ can be expanded in a Fourier cosine series

$$j(\nu(\theta)) = j_o + \sum_{k=1}^{\infty} j_k \cos k\theta \tag{14}$$

$$j_k = 2\langle j(\nu(\theta)) \cos k\theta \rangle_\theta. \tag{15}$$

Write (15) in the form

$$j_k = \frac{2}{\pi k} \int_a^b j(\nu)' \sin k\theta \, d\nu. \tag{16}$$

It follows that the Fourier coefficients of ν are

$$\nu_k = \frac{2}{\pi k} \int_a^b \sin k\theta \, d\nu. \tag{17}$$

From (3) it follows that ν_k is also given by

$$\nu_k = q_- \delta_{k1} - r j_k \qquad (k = 1,2,\ldots), \tag{18}$$

which is preferable to (17) for numerical work because it automatically becomes exact as $j_k \rightarrow 0$. The moments of $\nu(\theta)$ can be obtained in the same way as (10)

$$\langle \nu(\theta)^k \rangle_\theta = a^k + \frac{k}{\pi} \int_a^b \nu^{k-1} \, \theta(\nu) d\nu. \tag{19}$$

Define a *normalized waveform*

$$\phi(\theta) = (\nu(\theta) - q_+)/q_- \tag{20}$$

which has the property of reducing to the input waveform $\cos\theta$ whenever $j(\nu) = 0$. If $j(\nu)$ is a rapidly increasing function of ν (such as the exponential characteristic of a junction), it may be that the nonlinearity of $j(\nu)$ can be neglected over part of the cycle while over the rest of the cycle the nonlinearity effectively clamps ν at a constant upper limit. This is one-sided abrupt clipping. It is customary to define the *clipping factor* Φ in terms of the ratio of the true analog peak to the clipped peak as follows

$$\Phi = -20 \log \phi_{\max}, \tag{21}$$

where from (20)

$$\phi_{\max} = \phi(0) = (b - q_+)/q_-. \tag{22}$$

The power delivered to the load at frequency $k\omega$ is

$$p_k = \nu_k^2/2r. \tag{23}$$

A common way of specifying nonlinear distortion is in terms of the ratios

$$d_k = p_k/p_1 \qquad (k = 2,3,\ldots) \tag{24}$$

and

$$d = \sum_{k=2}^{\infty} d_k = (2\langle \nu^2\rangle_\theta/\nu_1^2) - 1. \tag{25}$$

We define the kth *harmonic distortion* by

$$D_k = 10 \log d_k, \tag{26}$$

and the total harmonic distortion by

$$D = 10 \log d. \tag{27}$$

III. CLIPPING MODEL

Assume that the distortion may be represented as abrupt clipping of the positive peaks. Then

$$\nu(\theta) = w \cos(\theta,\tau) - w\langle \cos(\theta,\tau)\rangle_\theta, \tag{28}$$

where w and τ are parameters to be determined, and

$$(\theta,\tau) = \begin{cases} \tau & 0 \leqslant \theta \leqslant \tau \\ \theta & \tau \leqslant \theta \leqslant \pi \end{cases}. \tag{29}$$

A simple calculation gives

$$\langle \cos(\theta,\tau)\rangle_\theta = (\tau \cos \tau - \sin \tau)/\pi. \tag{30}$$

The clipping factor is

$$\Phi = -20 \log(\cos \tau). \tag{31}$$

The Fourier coefficients are now obtained without numerical integration from the relations

$$\begin{aligned} \nu_1 &= (w/\pi)(\pi - \tau + \frac{1}{2} \sin 2\tau) \\ &= w + \ldots \end{aligned} \tag{32}$$

$$\begin{aligned} \nu_k &= [2w/\pi(k^2 - 1)] (\cos k\tau \sin \tau - k^{-1} \cos \tau \sin k\tau) \qquad (k = 2,3,\ldots) \\ &= -(2w/3\pi)\tau^3 + \ldots (k < \tau^{-1}), \end{aligned} \tag{33}$$

and the total distortion from

$$\begin{aligned} \langle \nu^2\rangle_\theta &= (w^2/2\pi)(\pi + \tau \cos 2\tau - \frac{1}{2} \sin 2\tau) - w^2\langle \cos(\theta,\tau)\rangle_\theta^2 \\ &= (\nu_1^2/2)[1 + (4/15\pi)\tau^5 + \ldots]. \end{aligned} \tag{34}$$

For the determination of w, τ consider (11) in the form

$$\nu(\theta) = q_{-} \cos \theta + q_{+} - rj(\nu). \tag{35}$$

Comparing (28) with (35) shows that

$$q_{-} = w, \qquad q_{+} = -w \langle \cos (\theta, \tau) \rangle_{\theta}, \tag{36}$$

and that $j(\nu)$ is being approximated by

$$j = \begin{cases} r^{-1} (\cos \theta - \cos \tau) q_{-} & 0 \leqslant \theta \leqslant \tau \\ 0 & \tau \leqslant \theta \leqslant \pi \end{cases}. \tag{37}$$

The normalized waveform defined in (20) is being approximated by

$$\phi(\theta) = \cos (\theta, \tau). \tag{38}$$

When the model is valid, it is also valid to neglect $j(a)$ in (7); it then follows from (7), (12) and (13) that w, τ must obey the relations

$$w = rg_1 \tag{39}$$

$$j(b)(1 - \cos \tau)^{-1} = g_1, \tag{40}$$

where

$$b = w[\cos \tau + (\sin \tau - \tau \cos \tau)/\pi]. \tag{41}$$

Equation (40) requires that (37) be exact at $\theta = 0$.

The clipping model is suitable for use with a minicomputer since no numerical integrations are required. If $j(b)$ is easy to evaluate, the iteration of (40) is easy to do by trial and error. The accuracy of the model is best determined by comparison with the results calculated by the method of Section II. However, it is possible to obtain a corrected waveform in the region of clipping by solving the implicit relation

$$j(\nu(\theta)) = (\cos \theta - \cos \tau) g_1 \quad (0 \leqslant \theta \leqslant \tau). \tag{42}$$

obtained from (37).

IV. RECEIVER SENSITIVITY

In Fig. 1 let the light power be

$$u = u_o + u_1 \cos \theta$$

$$\theta = \omega t, \qquad u_o \geqslant u_1 \geqslant 0. \tag{43}$$

The photodiode assumed here is a specific $n^+ \pi p^+$ silicon unit having the dark current characteristic shown in Fig. 2. It is typical of a class of photodetectors developed by H. Melchior[7] for lightguide applications not requiring the sensitivity of avalanche photodiodes.[10] It has a quantum efficiency at 900 nm of $\eta = 0.8$ and a very low series resistance

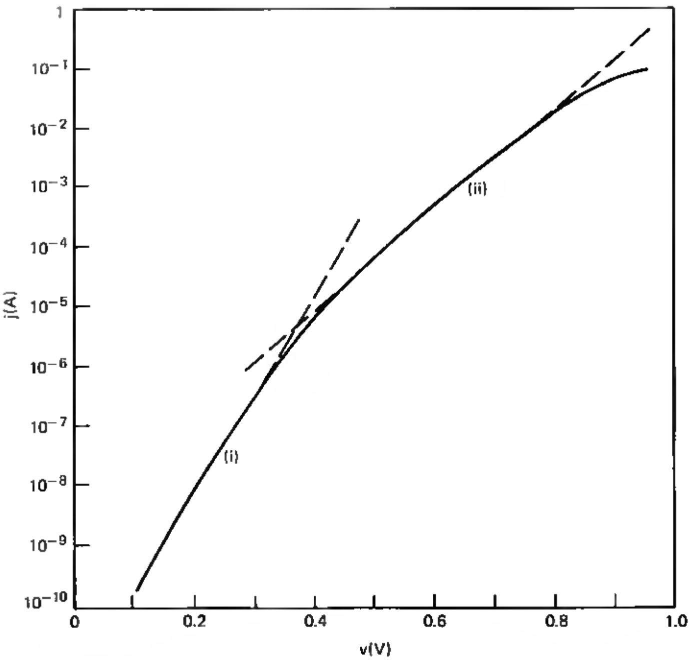

Fig. 2—Dark current-voltage characteristic of the photodiode. Parameters of the characteristic (44) for regions (*i*) and (*ii*) are listed in Table I.

$\approx 1\ \Omega$. The characteristic of Fig. 2 has two exponential regions of the form

$$j(\nu) = \alpha(e^{\beta\nu} - 1). \tag{44}$$

The current generator g in Fig. 1 can be written

$$g = \gamma u. \tag{45}$$

The parameters α,β,γ of the photodiode are listed in Table I.

The earphone is the ring armature telephone receiver[8] which provides an essentially flat response over the voice band, 300–3300 Hz. We shall specify this earphone by a resistance ρ and power sensitivity δ, both assumed independent of frequency over the band. The response may be

Table I — Photodiode parameters, $\eta = 0.80$, $\gamma = e\eta/h\nu = 0.58\ V^{-1}$ (at 900 nm)

	(*i*) $v < 0.37$ V	(*ii*) $v > 0.37$ V
α(A)	3.1×10^{-12}	3.8×10^{-9}
$\beta(V^{-1})$	38.70	19.35
$(\alpha\beta)^{-1}$ (Ω)	8.3×10^{9}	1.36×10^{7}

characterized by the relation

$$\text{SPL} = 81 + 10 \log (p/\delta) \qquad \text{(dB)}, \tag{46}$$

where SPL is the sound pressure level produced in a closed volume of 6 cm^3 and p is the inband power available from a matched generator of resistance ρ. Equation (46) shows that, when p equals the power sensitivity δ, the telephone receiver produces a speech pressure level (SPL) of 81 dB, which is the average level found in the telephone network by surveys.[11,12,13] Representative values for ρ,δ may be taken as

$$\rho = 128\ \Omega$$

$$\delta = 2.5 \times 10^{-7}\ \text{W}. \tag{47}$$

The transformer is assumed to be ideal over the band with primary taps to give a variable turns ratio n. The dc resistance is assumed negligible; this reduces the dc bias on the junction to zero, the most advantageous value. The inductance of the secondary must exceed 0.07 H determined by ρ and the low-frequency cutoff. It follows that the size, weight, and cost of the transformer would be approximately proportional to the maximum required value of n. In the following we specify the receiver by n; tbe effective ac load resistance of the photodiode is

$$r = n^2 \rho. \tag{48}$$

The sensitivity of a receiver of given n will be defined as tbe value of u_1 which produces SPL = 81 dB at the ear; thus the *sensitivity* s is defined by the relations

$$s = u_1, \qquad p_1 = \delta \tag{49}$$

where p_1 is defined by (23). There is a range of optical powers u_1 (and hence a range of sensitivities s) allowed by (49) because an increasing turns ratio n can be used to compensate (until nonlinearities become significant) for a decreasing optical power u_1. Figure 3 shows s plotted versus n. At the minimum we find by the method of Section II the values

$$n = 48, \qquad r = 0.29\ \text{M}\Omega\ (s = \chi) \tag{50}$$

and

$$\chi \equiv s_{\min} = 2.6\ \mu\text{W}. \tag{51}$$

The clipping model of Section III gives the same asymptotic straight line and the minimum value 2.5 μW.

The asymptotic straight line represents the sensitivity $\hat{s}$ of the *distortion-free receiver* that would result if we could take $j(\nu) \equiv 0$; the

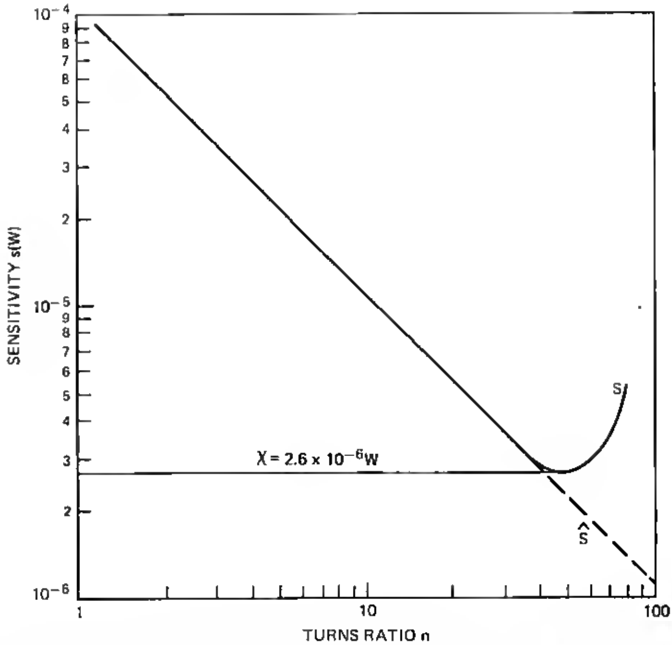

Fig. 3—Sensitivity s defined in (49) versus turns ratio n of transformer. The straight line shows the distortion-free receiver (52).

equation of the line is

$$\delta = \frac{\gamma^2 r \hat{s}^2}{2} = \frac{\gamma^2 n^2 \rho}{2} \hat{s}^2 \qquad \text{(distortion–free receiver).} \qquad (52)$$

V. RECEIVER HARMONIC DISTORTION

We define the *amplitude level* U_1 by

$$U_1 = 20 \log (u_1/\chi). \qquad (53)$$

The need for the factor of 20 in this equation in comparison to the factor of 10 in (46) results from (*i*) the photoelectric effect in the photodiode by which the electrical signal power produced is proportional to the square of the optical power modulation u_1 and (*ii*) the desire to express U_1 on the same logarithmic scale as SPL. In analogy to the present metallic telephone network we assume that the level reaching any receiver can be regarded as a random variable with a distribution[11,12,13] that is approximately normal with a variance of 7.8 dB. We define the *dynamic range* Γ of the optical channel

$$\Gamma \equiv U_o - \langle U_1 \rangle_N, \qquad (54)$$

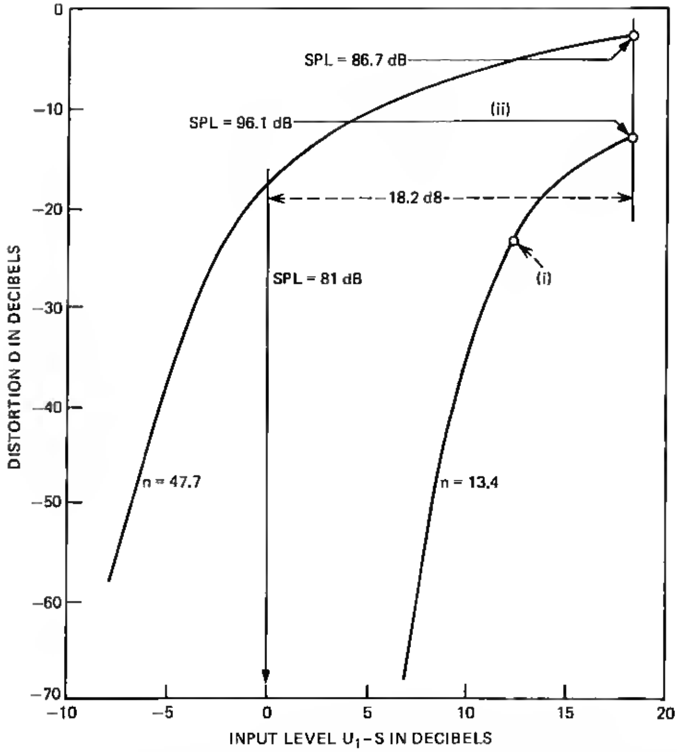

Fig. 4—Total harmonic distortion D defined in (27) versus input signal level U_1 defined in (53) relative to sensitivity level S defined in (56) for two values of n. Various sound pressure levels SPL are indicated; for all n, $U_1 = S$ corresponds to SPL = 81 dB. Points (i) and (ii) are chosen for waveform examination in Fig. 5.

where $\langle U_1 \rangle_N$ is the average over the level distrihution N (see Appendix A), and

$$U_o \equiv 20 \log (u_o/\chi) \tag{55}$$

is the *clipping level* of the channel. This will always be a bottom-side clipping level; we will also assume for simplicity that it is a top-side clipping level. We assume Γ is a system constant maintained hy the electrical-optical interfaces, whereas U_o and $\langle U_1 \rangle_N$ fall off with the transmission distance x of the particular lightguide. The dynamic range determines the probability that clipping will not occur, which we here call the *quality.* A more general discussion of Γ and speech quality is given in Appendix A.

Harmonic distortion $D_2, D_3, \ldots$ and total harmonic distortion D are defined in (26) and (27) respectively. These quantities are functions of U_1 and are only meaningful up to $U_1 = U_o = \Gamma + \langle U_1 \rangle_N$. We define a

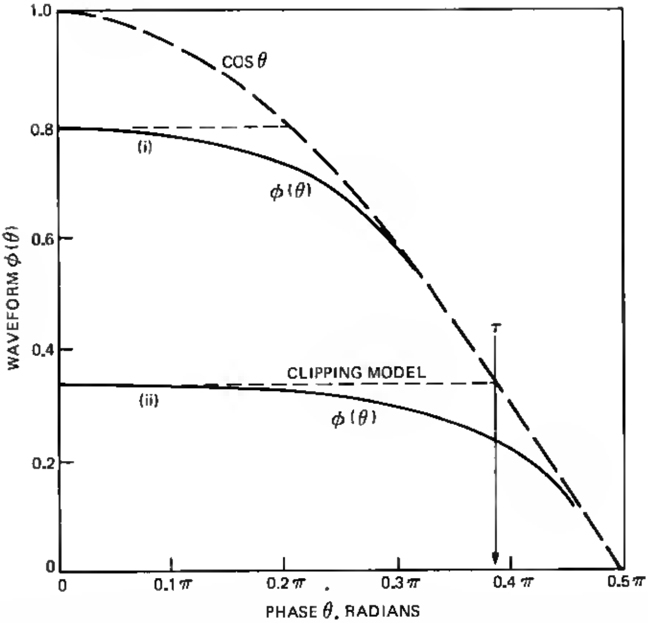

Fig. 5—Normalized waveform (20) for points (*i*) and (*ii*) of Fig. 4 for half of fundamental range of θ. The clipping model approximation is shown dashed and the parameter τ is indicated on (*ii*).

sensitivity level

$$S = 20 \log (s/\chi) \geqslant 0 \tag{56}$$

with a similar definition for $\hat{S}$ of the distortion-free receiver (52). For our calculations we have chosen to consider the receiver at the *reference point* $\overline{x}$ defined by

$$\langle U_1(\overline{x}) \rangle_N = S, \tag{57}$$

or the distortion-free receiver at the reference point $\overline{x}_{DF}$ defined by

$$\langle U_1(\overline{x}_{DF}) \rangle_N = \hat{S}. \tag{58}$$

Since we will presently conclude that significant distortion in the receiver will be avoided in practice, the distinction between $\overline{x}$ and $\overline{x}_{DF}$ need not concern us further. For illustrative purposes we choose the value $\Gamma = 18.2$ dB. The reasonableness of this value in terms of speech quality is discussed in Appendix A. Figure 4 shows D versus $U_1 - S$ for two receivers $n = 47.7$ and $n = 13.4$ out to $U_1 - S = 18.2$. The steep rise of these curves and the absence of any extensive straight portions show that D is not dominated by the second harmonic but involves a large number of harmonics. At several points the SPL at the fundamental is indicated to show

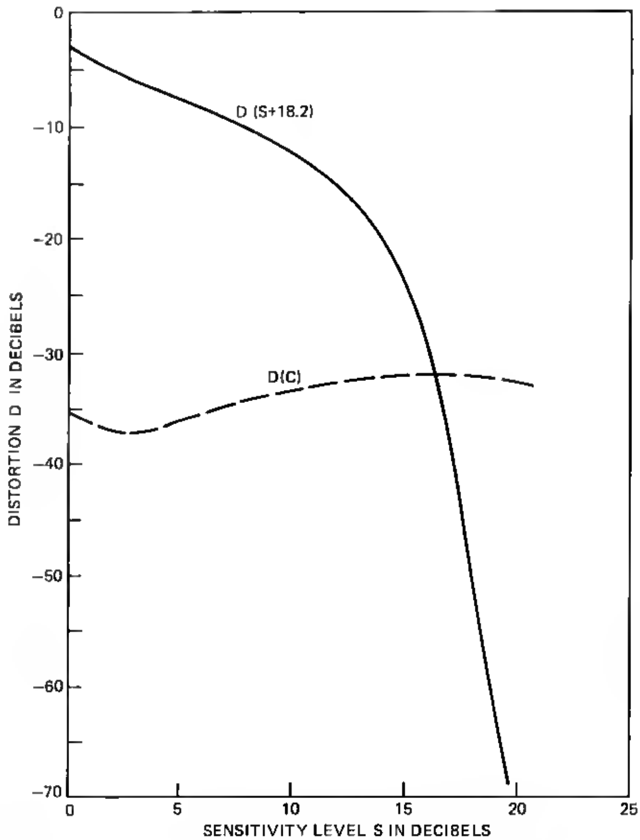

Fig. 6—Total harmonic distortion D $(S + 18.2)$ at the channel clipping level for the case $\Gamma = 18.2$ dB versus sensitivity level S defined in (56). Dashed curve shows $D(C)$ at receiver clipping level C defined in (61).

that an 18-dB rise in level does not produce a corresponding rise in SPL when D is high. For a distortion-free receiver, (46) becomes with the help of (52), (53), and (56)

$$\widehat{\mathrm{SPL}} = 81 + U_1 - \hat{S} \qquad \text{(distortion–free receiver).} \qquad (59)$$

The normalized waveform (20) is shown in Fig. 5 for two cases identified as points (i), (ii) in Fig. 4. The distortion of the waveform is shown to be a gradual clipping of the positive peak. In the clipping model this is approximated by abrupt clipping out to $\theta = \tau$ as indicated for curve (ii). We have obtained good results in calculating D_2, D_3 and D by the clipping model when $D > -40$ dB; higher harmonics are given with diminishing accuracy.

Figure 6 shows the total distortion at the channel clipping level

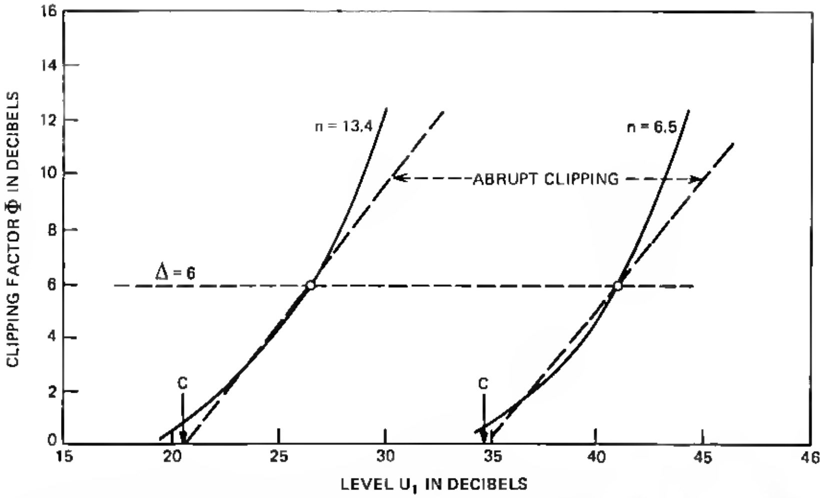

Fig. 7—Clipping factor (21) versus level (53) for two illustrative values of n. Dashed lines represent abrupt clipping approximation (61) which defines the clipping level C of the receiver. The clipping discount Δ (see Appendix A) is chosen as $\Delta = 6$ dB.

$D(S + 18.2)$ versus S for a receiver at the reference point. Conceivably an optimization could be based on an upper-limit objective for D at the channel clipping level. The dashed curve shows $D(C)$ at the receiver clipping level (to be defined in the next section). Notice that $D(C)$ is approximately constant, so in the present instance the quality matching principle is in effect equivalent to requiring $D < -32$ dB at the clipping level.

VI. RECEIVER CLIPPING LEVEL

The clipping factor has been defined in (21) for the general analysis and in (31) for the clipping model. The validity of the clipping model has been confirmed from the waveform and from calculations of D. Using the clipping model, we have calculated $\Phi(U_1)$ for various receivers n. (For brevity, n is not explicitly indicated in writing Φ.) Figure 7 shows the results for $n = 13.4$ and $n = 6.5$. It is known[14] that telephone speech quality, as determined in subjective listening tests, is not degraded appreciably by small clipping, $\Phi < \Delta$, where we call Δ the *clipping discount* and adopt the value

$$\Delta = 6 \text{ dB}. \tag{60}$$

(This value has been deduced by us from an examination of the unpublished work of A. M. Noll.[14]) The discount is shown as a dashed line in Fig. 7. At each intersection of the discount with Φ we draw a line of

unit slope as shown. This represents *abrupt clipping*

$$\Phi(U_1) \to \begin{cases} 0 & U_1 < C \\ U_1 - C & U_1 > C \end{cases} \tag{61}$$

at the *receiver clipping level* C. Thus C (as a function of n) is defined by the relation

$$\Phi(C + \Delta) = \Delta. \tag{62}$$

Notice that C is not very sensitive to the value chosen for Δ; any value of Δ in the range 2 to 8 dB would give about the same $C(\pm\ 1\ \text{dB})$.

If the concept of a clipping level is valid, the actual $\Phi(U_1)$ for the receiver can be replaced with the abrupt clipping approximation (61). A test of the validity of the concept is the distortion D at $U_1 = C$; for abrupt clipping D would be zero up to $U_1 = C$. Figure 6 shows $D(C)$ versus S; it is approximately constant around -35 dB. The departure of $\Phi(U_1)$ from abrupt clipping above Δ is not of great significance, because the important question in quality determination is whether degradation occurs, not how much degradation has occurred.

VII. QUALITY MATCHING

The *optimum sensitivity* is the smallest value consistent with the requirement that the quality of the channel not be degraded by the receiver. This gives the principle of *quality matching* expressed by the relation

$$C = U_o \qquad \text{(quality matching)}, \tag{63}$$

that is, the equality of receiver and channel clipping levels. This matching is to hold at all points x in the optical loop system.

Figure 8 shows a schematic diagram of levels in a system versus the distance x from the optical source at the electrical-optical interface. We define the power *transmission loss* of the lightguide in the usual way

$$\text{TL}(x) = 10 \log\,[u(0)/u(x)]\ \text{(dB)}. \tag{64}$$

At $x = 0$ the interface injects the optical power

$$u(0) = h_o + h_1 \cos\theta \tag{65}$$

at the levels

$$H_0 = 20 \log\,(h_o/\chi) = U_o(0)$$
$$H_1 = 20 \log\,(h_1/\chi) = U_1(0). \tag{66}$$

It follows that

$$U_o(x) = H_0 - 2\text{TL}(x)$$
$$U_1(x) = H_1 - 2\text{TL}(x). \tag{67}$$

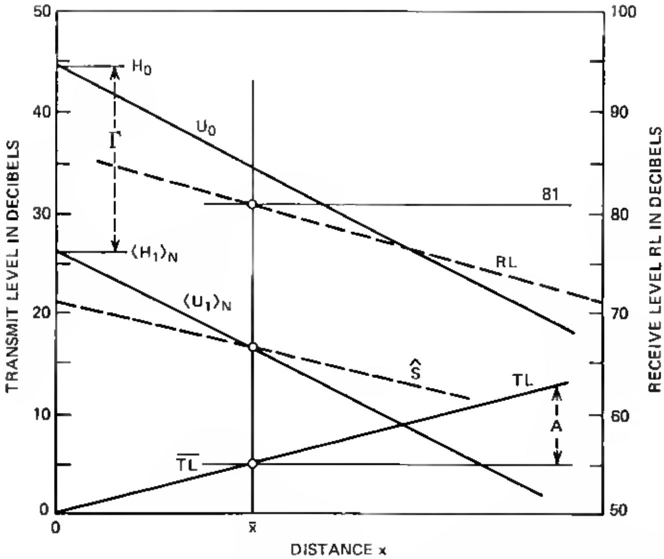

Fig. 8—Levels versus distance in a quality matched lightguide loop system (receiver portion only). The transmit levels (referring to the optical source) are the channel clipping level U_o, mean amplitude level $\langle U_1 \rangle_N$, and transmission loss TL defined in (55), (54), and (64) respectively. Receiver sensitivity level $\hat{S}$ is also shown on the transmit scale. Receive level (68) is also shown. Reference point $\bar{x}$ is defined in (57). The optical source is characterized by H_o; $\langle H_1 \rangle_N$ is defined in (66).

In Fig. 8, TL, U_0, and $\langle U_1 \rangle_N$ are called transmit levels and are referred to the scale on the left. Also shown referred to the left scale is the receiver sensitivity level S. The *receive level* $\mathrm{RL}(x)$ defined by

$$\mathrm{RL}(x) \equiv \langle \mathrm{SPL}(x) \rangle_N \tag{68}$$

is shown referred to the scale on the right. The reference point defined by (57) is denoted by $\bar{x}$. The dynamic range Γ defined by (54) is a constant of the system.

From (54), (57), and (63) we obtain the quality matching relations

$$C(\bar{x}) - S(\bar{x}) = \Gamma \tag{69}$$

$$C(\bar{x}) - C(x) = 2A(x), \tag{70}$$

where

$$A(x) \equiv \mathrm{TL}(x) - \mathrm{TL}(\bar{x}). \tag{71}$$

Figure 9 shows C, $C - S$, and $\hat{S}$ plotted versus n. The implementation of quality matching is illustrated for the case $\Gamma = 18.2$ dB. We find point (i) on the $C - S$ curve according to (69), which determines point (ii) on $\hat{S}$ and (iii) on C. The optical power at the source is then determined by

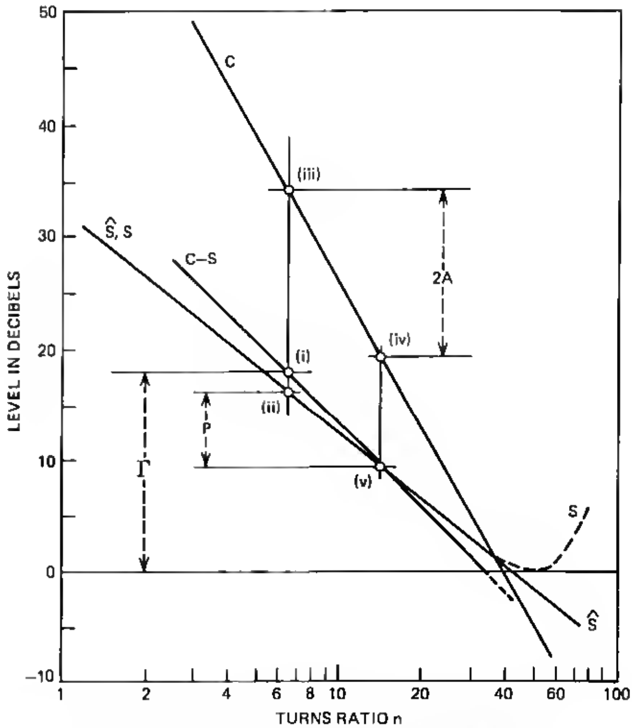

Fig. 9—Receiver clipping level C defined in (61) versus n. Also shown are $C - S$, $\hat{S}$, and S, where S is the sensitivity level (56) and $\hat{S}$ is the sensitivity level for the distortion-free receiver defined by (52). Points (i) to (v) trace the quality matching procedure based on (54), (63), (69), (70), and (74).

(iii) through the relation

$$H_o = C(\bar{x}) + 2\,\overline{\mathrm{TL}} \tag{72}$$

in terms of $\overline{\mathrm{TL}} \equiv \mathrm{TL}(\bar{x})$. The choice of $\overline{\mathrm{TL}}$ determines the average RL of the system, which is a system objective (not necessarily 81 dB) we must leave open at this time. We find point (iv) on C according to (70), which determines the optimum value of n for a receiver at x. It also determines point (v) on $\hat{S}$. (The intersection with S, when it differs from $\hat{S}$, is not of interest.) From (22) and (v) we determine the quantity

$$P(x) \equiv \hat{S}(\bar{x}) - \hat{S}(x). \tag{73}$$

From (59), (67), and (68) the receive level for a distortion-free receiver is

$$\widehat{\mathrm{RL}}(x) = 81 + P(x) - 2A(x). \tag{74}$$

This is an excellent approximation for RL, because quality matching guarantees that distortion is too small to have any effect on response.

In Fig. 9 the lines C and $\hat{S}$ are given by the relations

$$C = 70.5 - 44.3 \log n \tag{75}$$

$$\hat{S} = 32.6 - 20 \log n. \tag{76}$$

If $\Gamma > 5$ dB, it is justified to approximate

$$C - S = 37.9 - 24.3 \log n. \tag{77}$$

It follows that

$$C(\overline{x}) = 1.4 + 1.82\,\Gamma \tag{78}$$

$$P(x) = 0.90\,A(x) \tag{79}$$

$$\log n = 1.56 - 0.041\,\Gamma + 0.045\,A. \tag{80}$$

Thus the receive level is

$$\mathrm{RL} \approx \widehat{\mathrm{RL}} = 81 - 1.1\,A. \tag{81}$$

and the clipping level at the optical source is

$$H_o = 1.4 + 1.82\,\Gamma + 2\overline{\mathrm{TL}}. \tag{82}$$

VIII. SUMMARY AND DISCUSSION

We have obtained a practical algorithm suitable for a large computer for solving the nonlinear integral equation (3) referring to Fig. 1. The equation implies an integral restraint (4) on the solution which is an unusual feature that removes it from the types found discussed in texts on nonlinear networks. The load r is taken as zero at dc and a constant resistance at all signal frequencies of interest. This implies that the load is dispersive at frequencies below a certain cutoff frequency (300 Hz in the receiver problem). Ordinarily a nonlinear dispersive circuit requires nonlinear differential equations to describe it. Here we have avoided the differential equation and obtained instead an integral equation (3) by: (*i*) asking only for the periodic solution, and (*ii*) treating separately the ac and dc voltages with different values of r. The assumption of zero dc resistance is convenient and usually appropriate in practice, but the analysis presented in Section II can easily be generalized to any value of dc resistance. The nonlinear conductor $j(\nu)$ is passive ($j(0) = 0$) and monotonically increasing ($j(\nu)' \geqslant 0$) but otherwise arbitrary. The conditions on $j(\nu)$ rule out negative-resistance instabilities and guarantee a unique solution.

The method presented in Section II is exact in principle; it is based on the fact that any functional of the solution (e.g., a Fourier coefficient) can be calculated explicitly by integration once two parameters (a,b) have been determined from the implicit relations (12), (13). Standard

routines are available for solving simultaneous implicit relations to any desired precision. In using this method we have usually obtained satisfactory convergence with no special precautions. When convergence problems are encountered, the answer is to start the iteration with better estimates for a,b.

The clipping model described in Section III was originally worked out to provide initial estimates of a,b for use in the exact method. It soon became apparent, however, that it is sufficiently accurate in the receiver problem for all calculations of sensitivity, total distortion (when $D > -40$ dB), and clipping factor. The reason for this is that the exact waveform comes close to the clipped waveform assumed in the model. The clipping model is not recommended for the calculation of specific harmonics higher than the second. Generally the clipping model is expected to be useful whenever $j(\nu)$ is a rapidly increasing function. In this model distortion (clipping) is represented by a single parameter τ which is determined from the implicit relation (40). No numerical integrations are involved in using the model, which makes it convenient for use with a minicomputer.

The analysis of the photovoltaic receiver in the remainder of the paper is based on a sinusoidal input waveform. The calculation of the sensitivity s, defined in (49) as a measure of response based upon producing a certain reference sound level at the fundamental of 81 dB, is presented in Section IV. The photodiode and the earphone assumed in this calculation are the best presently available for the purpose. The transformer secondary must have an inductance of at least 70 mH, so the turns ratio n is adjusted by means of taps on the primary. By adjusting n, any value of s down to the minimum 2.6 μW can be obtained. However, the size and cost of the transformer are expected to increase approximately as the maximum value of n.

The minimum $s = \chi$ shown in Fig. 3 is a nonlinear effect having nothing to do with the impedance matching concept of linear circuit theory. The small signal resistance of the junction is given in Table I, $(\alpha\beta)^{-1} = 8.3 \times 10^9\ \Omega$ which is larger than $r(\chi) = 2.9 \times 10^5\ \Omega$ by a factor $\approx 3 \times 10^4$! This shows the necessity for a nonlinear analysis. For $n > 48$ the distortion increases very rapidly at the reference level assumed for the calculation of s. This does not mean, however, that $n > 48$ is ruled out for receivers operating at much lower levels. In a properly designed system the receiver must respond almost like the distortion-free receiver defined in (52) at the levels to which it is subjected. The reference level SPL = 81 dB is the overall average level for receivers in the existing telephone system.

At a particular point, the reference point, in a loop system the receive level defined in (68) will equal 81 dB, and at other points its value will depend on the transmission loss from the reference point to that point.

For a discussion of total harmonic distortion D we have considered a receiver at the reference point in Section V. We find (Fig. 4) that D rises very rapidly witb level up to about $D \approx -20$ dB and then begins to bend over. The value of D of interest is the value at tbe channel clipping level shown versus sensitivity in Fig. 6 for an assumed dynamic range of 18.2 dB. The results confirm that D is much too high in the minimum sensitivity receiver for use at the reference point. It is conceivable that some criterion on D (e.g., $D < -20$ dB) could be used as the basis for optimizing the receiver (choosing n). However, we believe that clipping provides a more objective and less arbitrary basis for optimization. The existence of clipping is shown by the waveform of Fig. 5 and by the good agreement between the clipping model and the exact method of D.

The clipping factor defined in (21) has been calculated as a function of level in Section VI. At a certain value Δ, called here the clipping discount, abrupt clipping begins to degrade speech quality.[14] Therefore an abrupt clipping approximation has been fitted to tbe receiver clipping factor at the value Δ. Tbis defines the clipping level C as illustrated in Fig. 7 for the choice $\Delta = 6$ dB. Actually C is not very sensitive to Δ. The total distortion at level C would be zero for abrupt clipping. We find $D(C) \approx -35$ dB for all receivers (Fig. 6); in our opinion this is small enough to confirm the validity of the clipping level concept.

Quality matching as an optimizing principle is defined in Section VII. It amounts to setting the channel and receiver clipping levels equal as in (63). Tbe quality of speech transmitted in an analog channel in which the only nonlinearity is abrupt clipping is determined by the dynamic range, defined for sinusoidal signals in (54). The dynamic range Γ is independent of lightguide transmission loss so we consider it a constant of the optical loop system. In Fig. 8 we pass from a strictly device viewpoint to a system viewpoint. The quality matching relations (69) determine the optical power needed at the reference point as well as the optimized receiver and receive level throughout the optical loop system. The various levels are shown schematically in Fig. 8 as a function of distance x from the optical source assuming the same source power for all loops. The curves required to obtain the solution are shown in Fig. 9, and approximate equations for the solution are given in (78) through (82).

In the existing system the mean loop insertion loss[15] is about 5 dB. The receive level defined in (68) is 81 dB at a point 5 dB from the central office, and 81 dB is approximately the mean loop receive level. This is not necessarily an objective for future loop planning, so we here emphasize that our analysis contains no such assumption. The use of 81 dB as a reference is only a convenience and involves no loss of generality. The reference point $\bar{x}$ in Fig. 8 with the transmission loss $\overline{TL}$ need not be the "average point" for the loop system. The mean loop receive level

is from (81)

$$\langle \text{RL} \rangle_L = 81 + 1.1\,(\overline{\text{TL}} - \langle \text{TL} \rangle_L) \tag{83}$$

where $\langle\ \rangle_L$ denotes an average over loops. This can be set at any desired level by properly choosing $\overline{\text{TL}}$.

The quality matching concept requires a receiver with adjustable turns ratio which probably involves some added cost compared with a fixed receiver. However, any fixed receiver would give a receive level varying as $-2A$,

$$\text{RL} = 81 - 2A \qquad \text{(fixed receiver)}, \tag{84}$$

whereas quality matching gives (81) varying approximately as $-A$. This effect is shown in Fig. 8 by the different slopes of RL and $\langle U_1 \rangle_N$. Clearly this is a desirable effect which utilizes the capabilities of the receiver to the fullest extent and permits serving a radius approximately twice that which would be possible with a fixed-sensitivity receiver having the sensitivity of an optimized receiver at the reference point. The actual range of tbe loop system would probably be limited by objectives on the minimum RL which we are not discussing here. Another limitation which we can only mention is that of transformer cost. From (80) we find that the cost, as measured by n, doubles for an increase of 6.7 dB in TL.

To illustrate the principles being discussed, we have chosen a realistic case specified by

$$\begin{aligned} \Gamma &= 18.2 \text{ dB} \\ \overline{\text{TL}} &= 5 \text{ dB} \qquad \text{(illustrative)} \\ A(x) &= 7.5 \text{ dB}. \end{aligned} \tag{85}$$

Points (i) to (v) in Fig. 9 trace the solution for this case. At the reference point $\overline{x}$ we find

$$\begin{aligned} C(\overline{x}) &= U_o(\overline{x}) = 34.6 \text{ dB} \\ u_o(\overline{x}) &= 0.14 \text{ mW} \\ s(\overline{x}) &= 17\ \mu\text{W}. \end{aligned} \tag{86}$$

At the point x we find

$$\begin{aligned} C(x) &= 19.6 \text{ dB} \\ n(x) &= 14.1 \\ \text{RL} &= 72.8 \text{ dB}. \end{aligned} \tag{87}$$

At the source we find

$$
\begin{aligned}
H_o &= 44.6 \text{ dB} \\
(H_1)_N &= 26.4 \text{ dB} \\
h_o &= 0.45 \text{ mW} \\
\text{peak source power} &= 2\,h_o = 0.9 \text{ mW}.
\end{aligned}
\tag{88}
$$

The fact that the peak source power is $2\,h_o$ follows from our assumptions of equality of top and bottom side clipping and of occurrence of bottom side clipping at $h = 0$.

The sensitivity at the reference level $s(\bar{x}) = 17\ \mu\text{W}$ is 50 times smaller (better) than that of the best optoacoustic receiver, the xenon photophone[4] ($s = 0.9$ mW). Furthermore, the photophone is a fixed receiver. Therefore the photovoltaic receiver is clearly superior for loop applications.

The "system" referred to here should be regarded as a relatively small subsystem of the loop plant serving a special class of customers wbo have lightguides running to their premises primarily to provide high capacity services. At the central office or other junction point there must be an electrical-optical interface containing the optical source for the receiver. The peak power of this source according to (88) should be 0.9 mW. This power into the lightguide is within the capabilities of present day heterojunction laser diodes[16] but about an order of magnitude above the capabilities of luminescent diodes.[16] To a limited extent the power at the source might be varied to compensate for the transmission loss; one could imagine all adjustment being done at the source instead of the receiver. Although we have described a system with fixed source power in Fig. 8 and believe that this is the most likely type of system, no change is required in the equations to treat the variable source. The dynamic range, however, would be determined by the interface circuitry and held to a uniform value to maintain transmission quality.

It may be objected that our nonlinear analysis has been based on a sinusoidal input signal whereas a telephone is required to transmit speech. Appendix A contains a discussion of the extension of the theory to a general waveform and the general definition of dynamic range. It is argued that the clipping level is valid for any waveform. A quality Q is objectively defined on the basis of the approximately normal distribution of speech levels in the telephone system. Figure 10 shows Q versus Γ for speech and for a hypothetical system having sinusoidal signals. Essentially, Q is a probability that speech is received without degradation caused by clipping. The illustrative case $\Gamma = 18.2$ dB used in this paper corresponds to $Q = 86$ percent and $Q_{\cos} = 99$ percent. An objectively defined quality is not equivalent to a grade of service determined sub-

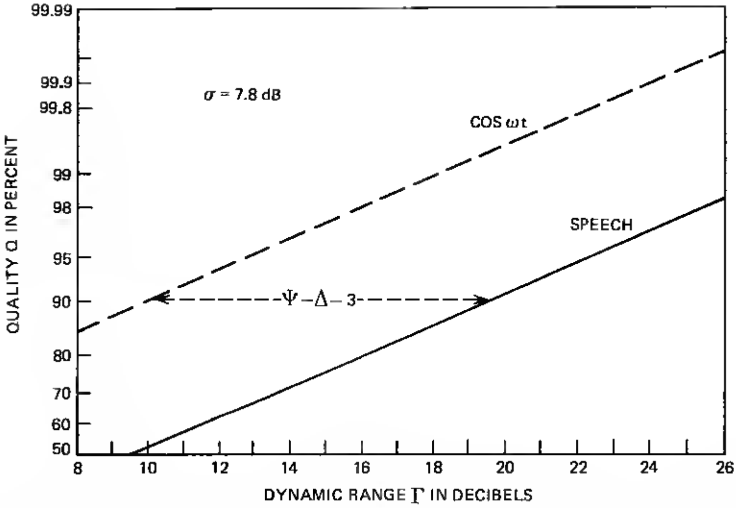

Fig. 10—Objective speech quality Q defined in (104) versus dynamic range Γ defined in (93) plotted on probability scale. The corresponding quantity for sinusoidal waveform is shown dashed.

jectively from listening tests, although we would expect a strong correlation hetween the two.

We shall not speculate on the circuitry of the interface, hut we offer an interesting ohservation in Appendix B. On the hasis of the light-emitting characteristic of laser diodes and the signals from the metallic network existing at the central office, it is shown that the direct application of the metallic network signal current (no amplifiers or transformers) to the laser would produce a somewhat greater level than $\langle H_1 \rangle_N$ called for in (88).

Let us suppose that $\overline{\mathrm{TL}} = 5$ dB in the illustrative case (85) is the average loss and the lightguide attenuation constant[17] is 2 dB/km. Then the reference point, which is also the average point, is at the distance

$$\bar{x} = 2.5 \text{ km} = 8.3 \text{ kft (illustrative)}. \tag{89}$$

This distance may be compared to the average loop length[15] 10.3 kft in the present metallic loop plant. A more conservative estimate of attenuation, like 6 dB/km, reduces (89) by a factor of three. Thus, the optical loop system seems to he limited to a somewhat smaller radius of service than the metallic loop system.

IX. ACKNOWLEDGMENTS

We acknowledge with thanks a crucial suggestion from D. A. Berkley on an earlier version of this work which caused us to change from the circuit we had first analyzed to the circuit of Fig. 1. We also thank H.

Melchior for valuable discussions and the use of unpublished data. We also thank W. M. Hubbard, D. Gloge, A. M. Noll, R. W. Dixon, D. Schinke, and S. D. Personick for important information and suggestions.

APPENDIX A

Speech Quality and Dynamic Range

We continue to let θ denote a normalized time variable, but in (43) and (65) we replace $\cos\theta$ with a more general normalized waveform $\psi(\theta)$

$$\begin{aligned} \max \psi(\theta) &= -\min \psi(\theta) = 1 \\ \langle \psi(\theta) \rangle_\theta &= 0. \end{aligned} \tag{90}$$

Define the *peak factor*

$$\Psi = -20 \log \psi_{rms} \tag{91}$$

and *signal level*

$$\begin{aligned} U &= 20 \log (u_1 \psi_{rms}/\chi) \\ &= U_1 - \Psi \end{aligned} \tag{92}$$

where U_1 is given by (53). Define the *dynamic range*

$$\begin{aligned} \Gamma &\equiv \langle 20 \log (u_o/2^{1/2} u_1 \psi_{rms}) \rangle_N \\ &= U_o - \langle U \rangle_N - 3 \end{aligned} \tag{93}$$

where the *channel clipping level* U_o is given by (55) and where

$$\langle U \rangle_N \equiv \int_0^1 U(N) dN, \tag{94}$$

N being a distribution function. For $\psi(\theta) = \cos\theta$ we have $\Psi = 3$ dB and (93) reduces to the definition first given in (54). For the distortion-free receiver the $\widehat{\text{SPL}}$ is

$$\widehat{\text{SPL}} = 81 + U - S + 3, \tag{95}$$

which reduces to (59) when $\Psi = 3$. We assume (61) and (63) remain valid but write (61) in terms of U

$$\Phi(U) = \begin{cases} 0 & U < U_o - \Psi \\ U + \Psi - U_o & U > U_o - \Psi. \end{cases} \tag{96}$$

The levels U are distributed[11,12,13] such that the probability that $U < X$ is

$$P(U < X) = N[(X - X_o)/\sigma] \tag{97}$$

where

$$X_0 = \langle U \rangle_N, \qquad \sigma = 7.8 \text{ dB} \tag{98}$$

and $N(x)$ is the normal distribution

$$N(x) = (2\pi)^{-1/2} \int_{-\infty}^{x} e^{-t^2/2}\, dt. \tag{99}$$

It is known from subjective studies[14] of the effect of clipping on telephone speech quality that no degradation occurs for $\Phi < \Delta$, where we take

$$\Delta = 6 \text{ dB}. \tag{100}$$

(This is a deduction from the work of A. M. Noll for which we assume responsibility.) The probability that $\Phi < \Delta$ is

$$\begin{aligned} P(\Phi < \Delta) &= N[(U_o + \Delta - \Psi - \langle U \rangle_N)/\sigma] \\ &= N[(\Gamma + \Delta + 3 - \Psi)/\sigma]. \end{aligned} \tag{101}$$

The speech waveform is characterized by the value[18]

$$\Psi = 18.6 \text{ dB} \qquad \text{(speech)}, \tag{102}$$

so that

$$\Delta + 3 - \Psi = -9.6 \text{ dB} \qquad \text{(speech)}. \tag{103}$$

The objective definition of quality is

$$\begin{aligned} Q &= 100 \times P(\Phi < \Delta) \qquad \text{(speech)} \\ &= 100 \times N[(\Gamma - 9.6)/7.8]. \end{aligned} \tag{104}$$

A corresponding definition can be given for sinusoidal signals having the same level distribution assuming $\Delta = 0$ and $\Psi = 3$

$$Q_{\cos} = 100 \times N(\Gamma/7.8). \tag{105}$$

Figure 10 shows Q and $Q_{\cos}$ versus Γ on a probability scale.

APPENDIX B

Comments on the Optical Source

A laser diode has a threshold current for lasing and at higher currents a steeply rising emission h as a function of current i. In the lasing region we assume[19]

$$\begin{aligned} dh/di &= \epsilon \\ \epsilon &= 2.3\ W/A. \end{aligned} \tag{106}$$

If a sinusoidal current of amplitude i_1 flows through the laser (superposed on a suitable bias current), the optical signal level H_1 defined in

(66) is

$$H_1 = 20 \log (\epsilon i_1/\chi). \tag{107}$$

Suppose that the current i_1 is that supplied to a matched load by a generator of resistance r and available power p; then (107) can be written

$$H_1 = 20 \log [2^{1/2}(\epsilon/\chi)(p/r)^{1/2}]. \tag{108}$$

Finally, suppose that the generator is the central office and p corresponds to the mean signal level; representative values are[20]

$$p = 2\,\mu\text{W}, \qquad r = 1166\,\Omega. \tag{109}$$

It then follows that the mean optical amplitude level at the source is

$$\langle H_1 \rangle_N = 34 \text{ dB} \qquad (h_1 = 0.13 \text{ mW}). \tag{110}$$

This compares very favorably with the level called for, 26 dB, in (88). This shows that the current flowing in a matching resistor (1166 Ω) combined with the "gain" of the laser emission characteristic provides more than enough signal without a matching transformer in the circuit.

If a matching transformer is used, r in (108) is replaced by the dynamic laser diode resistance[19,21] $r_d \approx 1.5\,\Omega$, giving $h_1 \approx 3.8$ mW. For a luminescent diode an appropriate value of ϵ in (106) is[16] $\epsilon \approx 5 \times 10^{-4}$ W/A giving for the transformer matched case with $r_d = 1.5\,\Omega$ the power $h_1 \approx 0.8\,\mu$W, $\langle H_1 \rangle_N = -10$ dB.

REFERENCES

1. I. Jacobs, "Lightwave Communications Passes Its First Test," Bell Laboratories Record, *54*, No. 11 (December 1976), pp. 290–297.
2. A. R. Meier, "Bell Labs Unveils a Practical Optical Communications System," Telephony, August 9, 1976, pp. 48–54.
3. S. E. Miller, E. A. J. Marcatilli, and T. Li, "Research Toward Fiber Transmission Systems," Proc. IEEE, *61*, No. 12 (December 1973), pp. 1703–1751.
4. D. A. Kleinman and D. F. Nelson, "The Photophone—An Optical Telephone Receiver," J. Acoust. Soc. Amer., *59*, No. 6 (June 1976), pp. 1482–1494.
5. D. A. Kleinman and D. F. Nelson, "The Photophone—Physical Design," J. Acoust. Soc. Amer., *60*, No. 1 (July 1976), pp. 240–250.
6. D. F. Nelson, K. W. Wecht, and D. A. Kleinman, "Photophone Performance," J. Acoust. Soc. Amer., *60*, No. 1 (July 1976), pp. 251–255.
7. H. Melchior, private communication.
8. E. E. Mott and R. C. Miner, "The Ring Armature Telephone Receiver," B.S.T.J., *30*, No. 1 (January 1951), pp. 110–140.
9. Thomas A. Stern, *The Theory of Nonlinear Networks and Systems*, Reading, Mass.: Addison-Wesley, 1965, pp. 95–136.
10. H. Melchior, "Photodetectors for Optical Communications Systems," Proc. IEEE, *58*, No. 10 (October 1970), pp. 1466–1486.
11. K. L. McAdoo, "Speech Volumes on Bell System Message Circuits," B.S.T.J., *42*, No. 5 (September 1963), pp. 1999–2012.
12. F. T. Andrews and R. W. Hatch, "National Telephone Network Transmission Planning in the American Telephone and Telegraph Company," IEEE Trans. on Commun. Tech., *COM-19*, No. 3 (June 1971), pp. 302–314.

13. Staff of Bell Laboratories, *Transmission Systems forCommunications*, Winston-Salem, N.C.: Western Electric Company Technical Publications, 1970, 4th ed., Fig. 4–5, p. 74.
14. A. M. Noll, private communication.
15. P. A. Gresh, "Physical and Transmission Characteristics of Customer Loop Plant," B.S.T.J. *48*, No. 10 (December 1969), pp. 3337–3385.
16. H. Kressel, I. Ladany, M. Ettenberg, and H. Lockwood, "Light Sources," Physics Today, *29*, No. 5 (May 1976), pp. 38–47.
17. A. G. Chynoweth, "The Fiber Lightguide," Physics Today, *29*, No. 5 (May 1976), pp. 28–37.
18. Staff of Bell Laboratories, *Transmission Systems for Communications*, Winston-Salem, N.C.: Western Electric Company Technical Publications, 1970, 4th ed., p. 222.
19. R. W. Dixon, private communication.
20. Staff of Bell Laboratories, *Transmission Systems for Communications*, Winston-Salem, N.C.: Western Electric Company Technical Publications, 1970, 4th ed., Fig. 4–12, p. 84, and Fig. 4–5, p. 74.
21. R. L. Hartman and R. W. Dixon, "Reliability of DH GaAs Lasers at Elevated Temperatures," Appl. Phys. Lett., *26*, No. 5 (March 1975), pp. 239–242.